Effects of quenched randomness induced by car accidents on traffic flow in a cellular automata model

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In this paper we numerically study the impact of quenched disorder induced by car accidents on traffic flow in the Nagel-Schreckenberg (NS) model. Car accidents occur when the necessary conditions proposed by [Boccara *et al.*J. Phys. A **30**, 3329 (1997)] are satisfied. Two realistic situations of cars involved in car accidents have been considered. Model *A* is presented to consider that the accident cars become temporarily stuck. Our studies exhibit the "inverse-λ form" or the metastable state for traffic flow in the fundamental diagram and wide-moving waves of jams in the space-time pattern. Model *B* is proposed to take into account that the "wrecked" cars stay there forever and the cars behind will pass through the sites occupied by the "wrecked" cars with a transmission rate. Four-stage transitions from a maximum flow through a sharp decrease phase and a density-independent phase to a high-density jamming phase for traffic flow have been observed. The density profiles and the effects of transmission rate and probability of the occurrence of car accidents in model *B* are also discussed.

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I. INTRODUCTION

Cellular automata (CA) models have been proven to be an excellent tool for studying problems of traffic flow [1–3]. Owing to the space-time discrete update, CA models are easy to implement on a computer to perform real-time simulations, and they allow the flexibility to adapt complicated features observed in real traffic systems. From a theoretical point of view, these kinds of models, which belong to the class of one-dimensional driven lattice gases, are of particular interest. The features of driven lattice gases (DLG) have far-reaching consequences, because even the stationary state of the system is not described in the framework of standard equilibrium statistical mechanics [4]. Therefore, several theoretical and practical applications have improved the understanding of empirical traffic phenomena.

The first CA model for traffic flow which is able to reproduce the basic phenomena encountered in real traffic system, e.g., phase transition from a freely moving phase to a jamming phase, was proposed by Nagel and Schreckenberg (NS) [5]. The NS model is a minimal model in the sense that any further simplification of the model leads to unrealistic behavior. In the past few years, some mutations of the NS model have been suggested to describe the real traffic dynamics on a more detailed level, such as the hysteresis effects (or the metastable states) on a one-lane road [6–8], synchronized traffic flow [9,10], etc.

Within the framework of the CA model, traffic accidents have been studied recently [11–15]. With the help of the necessary conditions for the occurrence of car accidents proposed by Boccara *et al.* [11], simulations of the probability for car accidents to occur in the NS model have been offered. The relations of car accidents to traffic flow and stopped cars in the periodic system have also been studied [15]. But, in the process of simulations, car accidents do not really occur when the necessary conditions are simultaneously satisfied; these dangerous situations are calculated and considered to be the signal of the occurrence of accidents [13,14].

In many cases, the effects of quenched disorder on traffic in the NS model have been extensively investigated. Basically, there are two different types of quenched randomness which are associated with the cars (i.e., particles) [16–20] and the road (i.e., sites) [21–25], respectively. In a model, such cars or sites leading to quenched randomness are usually called defects which have different properties from the rest. In the first case, corresponding to randomness in the braking probability of drivers or speed limit $v_{\max}$, the defect particles may have a different time-independent braking probability or a smaller maximal velocity. Such defects are not localized in space, in contrast to those corresponding to sitewise disorder, where in a localized region certain parameters of the model take different values, e.g., by imposing a speed limit or increasing the deceleration probability.

In this paper, we present another quenched randomness associated with car accidents. Cars involved in car accidents can be considered as a defect and also lead to a local reduction of the capacity of the highway. Usually, traffic accidents can give rise to traffic jams. But, unlike the localized defect whose position is fixed on the road, defects of car accidents can happen anywhere and anytime, and even more than one car accident may take place on the road at the same time, therefore both where and when car accidents occur are random in real traffic systems. Also different from the defect particles, whether car accidents occur or not is determined by the dynamics of the model. According to results in Refs. [11,15], the "wrecked" cars determined by traffic flow and stopped cars can interrupt the traffic and lead to the reduction

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of traffic flow. Owing to the nonlinear relations between traffic flow and car accidents, the situations are very complex and should be investigated. As far as we know, traffic jams induced by accidents have not been theoretically studied until now, because all of the CA models developed so far have been designed to avoid collisions among cars. Therefore, there is an open question as to how traffic accidents affect traffic flow. In this paper, we will adopt the conditions proposed by Boccara *et al.* to determine whether car accidents happen or not [11], and extend the NS model to take into account the effects of quenched randomness associated with car accidents on traffic flow. Some features of the fundamental diagram in real traffic, such as the so-called "inverse-λ form" and a flat plateau (i.e., a density-independent flux), have been obtained, depending on various situations of cars involved in traffic accidents,

The present paper is organized as follows. In Sec. II, we present the definition of the occurrence of traffic accidents and the proposed models. The results of simulations are given in Sec. III. The last section is devoted to a summary.

II. MODELS AND DEFINITION OF CAR ACCIDENTS

Before we start with our considerations of car accidents, let us introduce the CA model established by Nagel and Schreckenberg to describe single-lane highway traffic [5]. The model is defined on a one-dimensional lattice of L sites with periodic boundary conditions. Every site can be either empty or occupied by a car with velocity $v=0,1,2,\cdots,V_{\max}$. Let d_E denote the number of empty cells in front of a car and N denote the number of cars on the road, thus car density ρ is N/L. The following steps in parallel are used for all cars. The first rule is acceleration. If the speed of a car is lower than $V_{\max}$, the speed is increased by 1; the second rule is deceleration due to other cars. If the speed is higher than d_E, then it is reduced to d_E. The third rule is randomization. The speed of a moving car is decreased randomly by one unit with a braking probability p. The fourth rule is that the car moves forward according to its new speed determined in rules 1–3.

Because of keeping a safe distance given in the second rule of update, car accidents do not happen in the basic NS model. However, in real traffic, car accidents often occur because of careless drivers who do not respect safety distances. More precisely, if the car ahead is moving, expecting its moving at the next time step, the careless driver has a tendency to drive as fast as possible and increases safety velocity given in the second rule of update by one unit with a probability p'. At the next time step, it will arrive at the position of the moving car ahead. If the moving car ahead is suddenly stopped, a collision between the cars happens. Let $x(i,t)$ and $v(i,t)$ denote the position and velocity of the ith car at time t, respectively. The probability for car accidents to occur is calculated according to the following three necessary conditions proposed by Boccara *et al.* [11]. The first condition is $d_E \leqslant V_{\max}$. The second condition is $v(i+1,t)>0$. The last condition is $v(i+1,t+1)=0$. When those conditions are simultaneously satisfied, car accidents occur with the probability p'. Although the probability for car accidents has been studied in the basic CA models [13–15], car accidents do not really happen. In the process of numerical simulations, the above three necessary conditions are only regarded as a dangerous situation and an indicator of a car accident occurring.

The extended NS model we will present is considered as follows. If these conditions are satisfied simultaneously, the car will hit the car ahead with the probability p', and then be stopped suddenly. When the car really collides with the car ahead, it becomes wrecked. Usually, the accident car either becomes temporarily stuck or is stopped there for a long time, depending on the realistic situations of cars involved in accidents, e.g., the extent of the damage to the accident cars, or the appearance of police cars. Thus, in this paper, we mainly consider two situations of accident cars. Model A describes the situation in which the accident cars stop there temporarily and begin to move forward freely after T time steps, where T means the time interval during which the accident car temporarily stops. When other cars reach the positions occupied by accident cars, they will stop until the accident cars ahead move forward. To keep the car density on the road unchanged, we hypothesize in the paper that the accident car is not deleted and is regarded as the usual one after the time interval T of temporary stay. Model B considers that the "wrecked" car will stay there forever. In model B, the accident car can be treated as a bottleneck in the road; the cars behind will pass through the site of the accident car with a transmission rate r. Different from the bottleneck, whose position is immobile, the accident cars may appear anywhere and anytime, so long as the conditions for the occurrence of accidents are met.

The extended CA model has five parameters: the speed limit $V_{\max}$, the stochastic braking probability p, the car density ρ, the time interval T of temporary stay (or transmission rate r), and the probability for car collisions p'. Obviously, the extended model returns to the basic NS model in the case of $p'=0$. In the simulations, the length of a site corresponds to 7.5 m on a real road, one automaton time step is 1 s, and the velocity unit is roughly 27 km/h. It is assumed that $V_{\max}=5$, which implies a maximum velocity of 135 km/h, just as occurs in the normal free-flow speed in real traffic. Data points of the fundamental diagram are obtained by averaging over 2000 time steps and 20 initial configurations and 20 stochastic seeds for the probability p' in the system of $L=1000$ after discarding transient values of 2000 time steps. The inhomogeneous initial configurations are obtained from stochastic distributions of the position and velocity of the cars, and the homogeneous configurations correspond to the cases in which the velocity of the cars and the interval between the nearest cars are uniform. In this paper, we mainly study the effects of car accidents on the traffic flow and ignore the braking probability p.

III. SIMULATION AND RESULTS

A. Model *A*

In this section, we discuss how the accident cars can move forward freely after the time interval of temporary stay. Let us consider the NS model with $V_{\max}=5$ in the case of

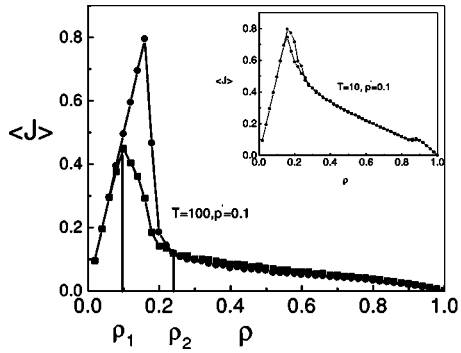

FIG. 1. The relation of the traffic flow $\langle J\rangle$ to the car density ρ in the system of $L=1000$ in the case of $T=100$ and $p'=0.1$. Circles correspond to the initial homogeneous configurations and squares correspond to the initial inhomogeneous conditions in model A. The metastable state in the interval $\rho_1<\rho<\rho_2$ can be seen, where $\rho_1=0.10\pm0.02$ and $\rho_2=0.24\pm0.02$. The relation of the traffic flow $\langle J\rangle$ to the car density ρ in the case of $T=10$ is shown in the inset.

$p=0$. The probability p' is set as 0.1. The fundamental diagram is calculated from an initial homogeneous configuration. As shown in Fig. 1, in the low-density region, where no car accident happens, cars move forward freely, therefore traffic flow $\langle J\rangle$ increases linearly as the car density ρ increases. In the high-density region, where the occurrence of car accidents delays movements of other cars behind, traffic flow becomes slower compared to the flow in the basic NS model without car accidents.

Particularly, discontinuous reduction of traffic flow at the critical density ρ_c is observed in Fig. 1, where $\rho_c=1/(1+V_{\max})$ is the critical density. Below the critical density, traffic flow reaches a maximum. Near the critical density where car accidents would occur because of careless drivers, once one car is stopped for a while due to car accidents, the cars behind will rapidly pile up, leading to a sharp decrease of the traffic flow. Thus, the "inverse-λ form" of the traffic flow in the fundamental diagram is observed, as shown in Fig. 1.

More importantly, traffic flow depends nonuniquely on car density, especially in the high-density region. As shown in Fig. 1, another curve in the fundamental diagram is obtained from an initial inhomogeneous configuration in the case of the same parameters. In Fig. 1, traffic flow increases linearly with the increase of car density in the very-low-density region. But at the density ρ_1, which is below the critical density ρ_c, traffic flow begins to depart from a linear increase, and even decreases with a further increase of car density, where ρ_1 represents the transition density from the freely moving phase to the jamming phase in the case of an initial inhomogeneous condition. In particular, as shown in Fig. 1, the metastable state appears not only in the low-density freely moving phase but also in the high-density jamming phase.

The metastable state in the interval $\rho_1<\rho<\rho_2$ in the fundamental diagram can be well explained as follows. Starting from an initial inhomogeneous configuration, the system will have some jams that are never smoothed out, due to the occurrence of car accidents. The steady state in this case is an inhomogeneous mixture of the jam-free region and higher-density jammed regions. Obviously, these jammed regions lower the average traffic flow, thus the lower branch of traffic flow corresponds to an initial inhomogeneous condition. As for the initial homogeneous conditions, below the critical density ρ_c, there is no car accident, thus no jam exists and traffic flow will still be a linearly increasing function of the density until the car density reaches the critical density ρ_c. Thus the upper branch corresponds to an initial homogeneous condition.

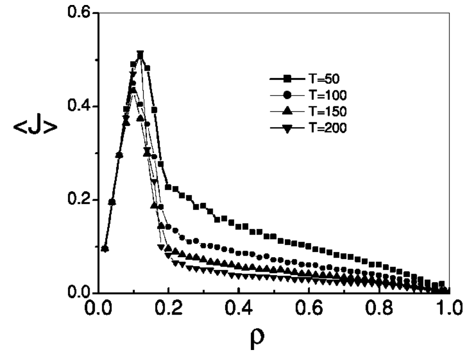

FIG. 2. The relation of $\langle J\rangle$ to the car density ρ in the system of $L=1000$ with various values of T calculated from the initial inhomogeneous configurations. The probability $p'=0.1$ in model A.

The density interval in which the metastable state of the traffic flow occurs shrinks with the decrease of the time interval T of temporary stay (shown in the inset of Fig. 1). The smaller the time interval T of temporary stay, the smaller the jams which never disappear completely due to car accidents, therefore leading to the increase of the critical density ρ_1 at which a transition from the freely moving phase to the jammed phase takes place and the decrease of the density ρ_2 at which no metastable states exist.

Figure 2 shows the fundamental diagram calculated from initial inhomogeneous configurations in the case of various values of time interval T of temporary stay. In Fig. 2, with the increase of the time interval T of temporary stay, the jammed regions which are not smoothed out due to car accidents become larger, leading to a decrease of the traffic flow, as expected in the high-density region.

The parameter p' denotes the probability for a car accident to actually happen when dangerous situations arise. When p' is small, fewer accidents occur in the system, leading to the small decrease of traffic flow in the region of high car density. In this case, due to the small fluctuation in the occurrence of car accidents, traffic flow shows a multipeak function of the car density near the critical density ρ_c (shown in Fig. 3). With the increase of p', the occurrence of car accidents results in a further decrease of the traffic flow. These numerical results are shown in Fig. 3.

Figure 4 shows a space-time diagram in which the extended CA model displays a moving wave of jam induced by accidents. If the accident cars are temporarily stuck, cars behind will pile up soon. While the wrecks are removed, they remain locked at a standstill because each driver is waiting for the car ahead to move. When the cars ahead leave, the cars still cannot accelerate instantly and must delay leaving

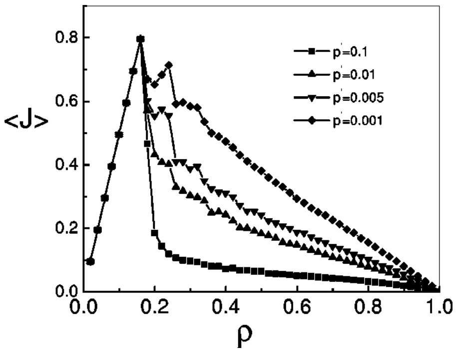

FIG. 3. The relation of $\langle J\rangle$ to the car density ρ in the system of $L=1000$ with various values of p' calculated from the initial inhomogeneous configurations. The time interval T of temporary stay is equal to 100 in model A.

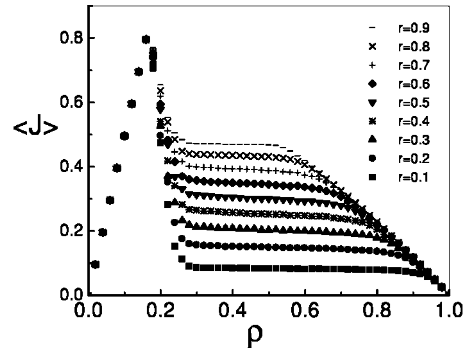

FIG. 5. The relation of $\langle J\rangle$ to the car density ρ in the system of $L=1000$ with various transmission rates r in the case of $p'=0.000\,03$ in model B.

for a moment because of not enough space in front of them. Each departing car must delay in the same way, and this causes the jam to "evaporate" starting from the forward downstream end (near the wreck).

While some cars are still jammed, more cars are piling up behind them at the trailing end of the jam. Even after the wreck is removed, more cars are still "condensing" onto the back of the jam. The traffic jam is like a solid object whose front end is evaporating and whose back end is growing like a crystal. The stoppage is creeping slowly upstream, in the opposite direction to the moving cars. The accident is gone, but a moving wave of stopped cars remains behind.

As shown in Fig. 4, since car accidents delay the movement of other cars behind, the density far downstream is lower than the density of maximum flow. The cars can move forward freely in the low-density region, where spontaneous formation of jams is highly unlikely. Therefore, the system exhibits the phase-separated steady state consisting of a macroscopic jam and a macroscopic free-flow regime, both of which simultaneously coexist.

Now we briefly discuss the hypothesis that the accident cars are not deleted from the system and are considered as the usual ones after the time interval T of temporary stay. In real traffic, if car accidents occur, police cars may appear and remove the wrecked cars. In this situation, we think that the

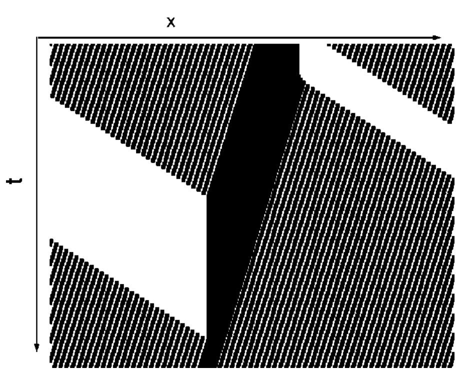

FIG. 4. A space-time diagram of model A with $\rho=0.16$, $T=50$, $p'=0.1$. Each horizontal row of dots represents the instantaneous positions of the cars moving right while the successive rows of dots represent the positions of the same cars at successive time steps.

"wrecked" cars are removed, new cars will enter the system, and can be considered as the usual ones. Thus, once car accidents happen on the road, the cars behind will pile up, leading to jams which will never disappear. Therefore, the main features in the system will not be changed distinctly. In another situation in which the accident cars are not deleted, but stagger forward at a speed slower than that of the usual ones after delaying several time steps, the jams formed due to the occurrence of car accidents do not disappear forever in the system when the car density is above the critical density ρ_1, because the accident cars move only in the region where the car density is lower than the density of maximum traffic flow. Thus such basic phenomena as the metastable state and the moving wave of jams could also be observed.

B. Model *B*

In this subsection, we discuss another situation of the "wrecked" car. When the car driven by the careless driver hits the car ahead, it will stay at the site forever. In this case, the "wrecked" car can be considered as the "defect," just as a bottleneck on the road in which other cars behind pass through the site with a transmission rate r. But, different from the bottleneck on the road whose position is fixed, the "wrecked" car determined by traffic flow can anchor at positions where the conditions for accidents to happen are met.

Let us consider the effects of the transmission rate r on the fundamental diagram. Figure 5 shows the $\langle J\rangle$-ρ curve obtained by averaging over 20 initial homogeneous configurations and 20 stochastic seeds of the probability for transmission rate in the system of $L=1000$ in the case of $p'=0.0003$. Four phases are recognized in Fig. 5. As expected, below the critical density ρ_c, no car accidents exist, therefore cars move forward freely, and traffic flow increases linearly with the increase of the density ρ. Near ρ_c, the traffic flow decreases sharply from the maximum traffic flow because of the block of car accidents. Above the critical density, traffic flow shows an approximate flat plateau in a certain interval of the car density, as do the features of the fundamental diagram in the NS model with a single "defect." And in the limit of high density, the traffic flow decreases linearly with the increase of the car density, like that of the NS model, owing to lower probability for accidents to occur.

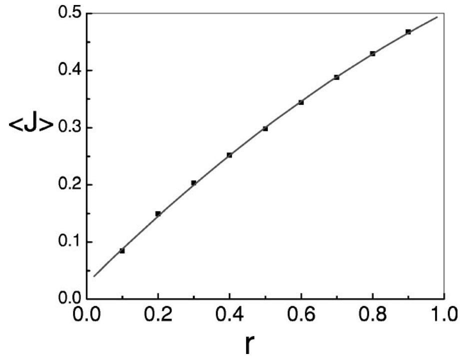

FIG. 6. The relation of the traffic flow in the region of a flat plateau to the transmission rate r in the case of $p'=0.000\,03$ in model B. Solid line is the result of expression (1).

With the increase of transmission rate, the value of the traffic flow in the flow-constant phase increases. The relation of $\langle J\rangle$ to r is shown in Fig. 6. By nonlinear fitting, the traffic flow can be approximately expressed as follows:

$$\langle J\rangle = a + br + cr^2, \tag{1}$$

where $a=0.028\pm0.004$, $b=0.62\pm0.02$, and $c=-0.15\pm0.02$. This is apparently different from the effects of the bottleneck studied previously [23]. We can also observe from Fig. 5 that the width of the flat plateau is decreased with the increase of the transmission rate r.

Different from the bottlenecks whose positions are fixed from the beginning, whether the "wrecked" cars appear or not is determined by traffic flow because of the nonlinear relations between the occurrence of car accidents and traffic flow [11,15]. If car accidents occur, the "wrecked" cars can reduce traffic flow, but traffic flow directly related to car accidents may result in the further occurrence of car accidents, therefore the fundamental diagram and the relations between $\langle J\rangle$ and r in our model B are different from those in models with bottlenecks, although the "wrecked" cars are also fixed bottlenecks.

To understand the complex relations between traffic flow and car accidents, we investigate the dynamic process of traffic flow. Figure 7 shows the relations of traffic flow to time during the initial time interval. As shown in Fig. 7, once a car accident occurs, traffic can be interrupted and the value of traffic flow is decreased because cars behind pass through the position occupied by the "wrecked" car with the transmission rate r. The decrease of the traffic flow cannot prevent cars from colliding with each other, and collisions between cars leads to a further decrease of traffic flow, until car accidents could not occur. As shown in Fig. 7, after nearly 800 time steps, traffic flow fluctuates around one level because car accidents no longer occur.

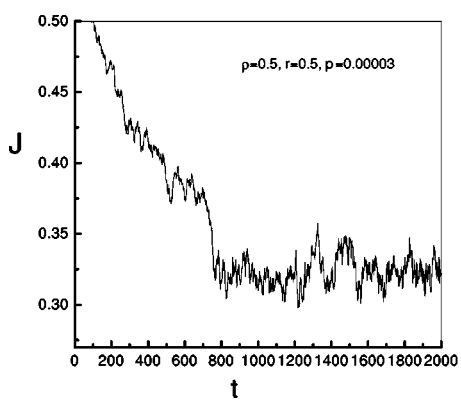

FIG. 7. The relation of traffic flow J to time t in model B with $\rho=0.5$, $r=0.5$, and $p'=0.000\,03$, during the initial time interval.

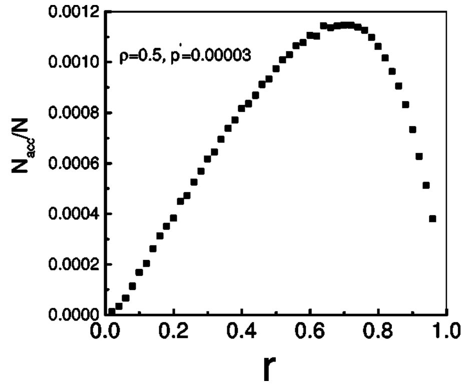

FIG. 8. The "wrecked" cars fraction $N_{\rm acc}/N$ as a function of transmission rate r. Car density $\rho=0.5$, and $p'=0.000\,03$.

Traffic flow can result in the occurrence of car accidents; conversely, car accidents can interrupt traffic and reduce traffic flow. Can an increase of traffic flow result in more car accidents, or can the occurrence of car accidents be suppressed only by a decrease of traffic flow? To grasp this question, we investigate the relationship between the number of car accidents and the transmission rate r. As shown in Fig. 8, with an increase of the transmission rate r, the number of "wrecked" cars increases linearly, reaches a maximum, and decreases with a further increase of r. The rate r for the maximum of the number of "wrecked" cars is known as the most probable rate, i.e., the rate at which car accidents occur most frequently.

According to previous results in Ref. [15], the probability of the occurrence of car accidents is directly related to traffic

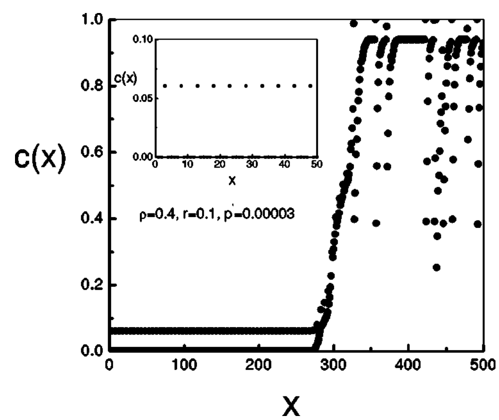

FIG. 9. Density profiles in model B with $\rho=0.5$, and $p'=0.000\,03$. The inset shows density profiles in the free-moving region.

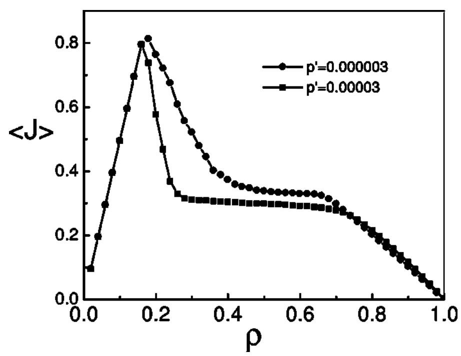

FIG. 10. The relation of $\langle J \rangle$ to the car density ρ in the system of $L=1000$ with various probability p' in the case of $r=0.5$ in model B.

flow and stopped cars. The probability for car accidents to occur increases with an increase of traffic flow, reaches a maximum value, and decreases with a further increase of traffic flow. In model B, an increase of transmission rate r can cause an increase of traffic flow (shown in Fig. 6). Therefore, the number of "wrecked" cars increases with an increase of rate r due to the small value of traffic flow. And the number of "wrecked" cars can decrease with a further increase of rate r because of the small number of stopped cars, although traffic flow is higher. These results also give us the indication that two alternative effective means for avoiding more car accidents may be adopted: the cars behind pass through the defects very slowly or very quickly.

Figure 9 shows the density profiles for different transmission rate r. Different from the density profiles in the models with bottlenecks [1,24], where phase segregation states with macroscopic high- and low-density regions have been identified, the density profiles show two different phases namely the free-moving phase and the jamming phase, respectively. As shown in Fig. 9, the "wrecked" cars distribute not uniformly but randomly in a region where the cars pile up. Unlike the high-density region in models with bottlenecks, density profiles in the jamming region exhibit the cracked high-density phase. In the free-moving phase, as shown in the inset in Fig. 9, car density can only be measured at the sites consisting of exact equal intervals, therefore cars move at the maximum speed.

In the opinion of Refs. [11,15], the production of "wrecked" cars is a self-adaptive process. Once a car accident occurs on the road, cars behind will pile up, and the stoppage will creep upstream in the opposite direction to traffic flow. In this case, a moving jam wave will collide with traffic flow and lead to additional car accidents, which result in a further heaping of cars behind. The process will not terminate until the increase in the local velocity of piling cannot compensate for the reduced traffic flow around "wrecked" cars, so that the cars can no longer pile up. Therefore, the "wrecked" cars are inclined toward congregating together, and in the free-moving region, cars move at the maximum speed.

Traffic flow is also affected by the probability p' for accidents to happen, because it directly determines how many accidents occur on the road when the conditions for the occurrence of accidents are met. Figure 10 shows the fundamental diagram in the case of various values of p' at $r=0.5$. As shown in Fig. 10, smaller values of the parameter p' lead to less change of traffic flow when the car density is above the critical density.

IV. CONCLUSION

In this paper, we investigated the effects of quenched randomness associated with car accidents on traffic flow. We adopt the conditions for the occurrence of car accidents proposed by Boccara *et al.*[11] to determine whether car accidents happen or not, and consider two realistic situations when car accidents occur. Model A considers that the accident cars have simply become temporarily stuck, and can move forward freely after the time interval of temporary stay. Model B supposes that the "wrecked "cars stand there forever, and other cars behind pass over the sites occupied by the "wrecked" cars with the transmission rate r. Using computer simulations, we find the metastable state or the "inverse λ form" of the traffic flow in the fundamental diagram due to the delay of car accidents, which have been observed from empirical investigations. A moving wave of jams formed in model A has also obtained. In model A, if the "wrecked" cars are weeded out after the time interval T of temporary stay, numerical simulations show the same phenomena.

In model B, car accidents lead to the four-stage transitions of traffic flow, from the maximum traffic flow phase through the sharp decrease phase and from the flat-plateau phase to the high-density jamming phase. The density profiles show two different phases, namely the free-moving phase and the jamming phase, respectively. The effects of the transmission rate on the traffic flow have been studied.

Unlike the localized defect whose position is fixed on the road, whether car accidents occur or not is determined by the dynamics of the model. Therefore, our results are different from those corresponding to models with bottlenecks.

As far as we know, traffic jams induced by car accidents have not really been studied until now, because all the models developed so far are designed to avoid collisions among cars. In addition, experimental studies of accident jams are impractical. Therefore, our models provide a possible approach to describe such traffic jams actually induced by car accidents.

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